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SOLVING FUZZY TRANSPORTATION PROBLEM USING SYMMETRIC TRIANGULAR FUZZY NUMBER

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ABSTRACT− *The transportation problem is one of the earliest applications of linear programming problems. In the literature, several methods are proposed for solving transportation problems in fuzzy environment but in all the proposed methods, the parameters are normal fuzzy numbers. In this paper, a general fuzzy transportation problem is discussed. In the proposed method, transportation cost, availability and demand of the product are represented by symmetric triangular fuzzy numbers. We develop fuzzy version of Vogel's algorithms for finding fuzzy optimal solution of fuzzy transportation problem. A numerical example is given to show the efficiency of the method.*

Keywords – Fuzzy sets, Symmetric Triangular Fuzzy numbers, Fuzzy transportation problem, Fuzzy ranking, Fuzzy arithmetic.

1, INTRODUCTION.

The Transportation problem is a special type of linear programming problem which deals with the distribution of single product (raw or finished) from various sources of supply to various destination of demand in such a way that the total transportation cost is minimized. There are effective algorithms for solving the transportation problems when all the decision parameters, i.e. the supply available at each source, the demand required at each destination as well as the unit transportation costs are given in a precise way. But in real life, there are many diverse situations due to uncertainty in one or more decision parameters and hence they may no be expressed in a precise way. This is due to measurement inaccuracy, lack of evidence, computational errors, high information cost, whether conditions etc. Hence we cannot apply the traditional classical methods to solve the transportation problems successfully. Therefore the use of Fuzzy transportation problems is more appropriate to model and solve the real world problems. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand are fuzzy quantities.

Bellman and Zadeh [2] proposed the concept of decision making in Fuzzy environment. After this pioneering work, several authors such as Shiang-Tai Liu and Chiang Kao[12], Chanas et al[1], Pandian et.al [11], Liu and Kao [10] etc proposed different methods for the solution of Fuzzy transportation problems. Chanas and Kuchta [1] proposed the concept of the optimal solution for the Transportation with Fuzzy coefficient expressed as Fuzzy numbers. Chanas, Kolodziejckzy, Machaj[4] presented a Fuzzy linear programming model for solving Transportation problem. Liu and Kao [10] described a method to solve a Fuzzy Transportation problem based on extension principle. Lin introduced a genetic algorithm to solve Transportation with Fuzzy objective functions.

Nagoor Gani and Abdul Razak [7] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. A.Nagoor Gani, Edward Samuel and Anuradha [6] used Arshamkhan's Algorithm to solve a Fuzzy Transportation problem. Pandian and Natarajan [11] proposed a Fuzzy zero point method for finding a Fuzzy optimal solution for Fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers.

In general, most of the existing techniques provide only crisp solutions for the fuzzy transportation problem. In this paper we propose a simple method, for the solution of fuzzy transportation problems without converting them in to classical transportation problems using Symmetric Triangular Fuzzy number.

2. PRELIMINARIES

The aim of this section is to present some notations, notions and results which are of useful in our further consideration.

2.1 Fuzzy numbers.

A fuzzy set A defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_A : R \to [0,1]$ has the following characteristics

(i) A is normal. It means that there exists an x $\in$ R such that $\mu_A(x) = 1$

(ii) A is convex. It means that for every $x_1, x_2 \in R$, $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$, $\lambda \in [0,1]$

(iii) μ_A is upper semi-continuous.

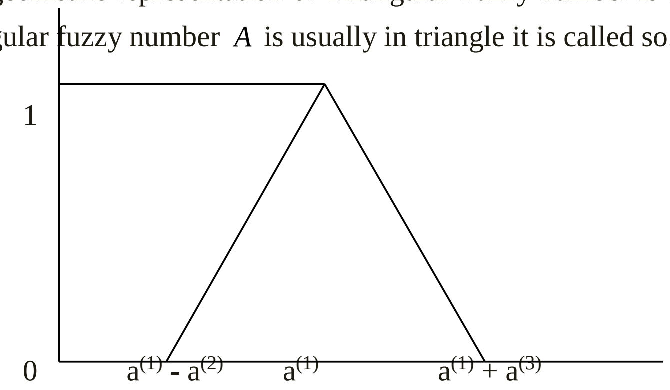
(iv) supp (A) is bounded in R.

2.2 Triangular Fuzzy number

A fuzzy number A in R is said to be a triangular fuzzy number if its membership function $\mu_A : R \to [0,1]$ has the following characteristics.

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1 \quad , & x = a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0 \quad , & otherwise \end{cases}$$

It is denoted by $A = (a^{(1)}, a^{(2)}, a^{(3)})$ where $a^{(1)}$ is Core (A), $a^{(2)}$ is left width and $a^{(3)}$ is right width. The geometric representation of Triangular Fuzzy number is shown in figure. Since, the shape of the triangular fuzzy number A is usually in triangle it is called so.



Membership function of triangular fuzzy number

The Parametric form of a triangular fuzzy number is represented by $A = \left[a^{(1)} - a^{(2)}(1-r),\ a^{(1)} + a^{(3)}(1-r) \right]$

2.3 Symmetric Triangular Fuzzy number

If $a^{(2)} = a^{(3)}$, then the triangular fuzzy number $A = (a^{(1)}, a^{(2)}, a^{(3)})$ is called symmetric triangular fuzzy number. It is denoted by $A = (a^{(1)}, a^{(2)})$, where $a^{(1)}$ is Core (A), $a^{(2)}$ is left width and right width of c.

The parametric form of a symmetric triangular fuzzy number is represented by $A = \left[a^{(1)} - a^{(2)}(1-r),\ a^{(1)} + a^{(2)}(1-r) \right]$

3. Ranking of Triangular Fuzzy number

Several approaches for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for comparing the fuzzy numbers is by the use of a ranking function based on their graded means. That is, for every $A = (a^{(1)}, a^{(2)}, a^{(3)}) \in$F(R), the ranking function $\Re$: F(R) $\rightarrow$ R by graded mean is defined as

$$\Re(A) = \left(\frac{a_1 + 4a_2 + a_3}{6}\right) \qquad (\because a_2 = a_3)$$

For any two fuzzy triangular Fuzzy numbers $A = (a^{(1)}, a^{(2)}, a^{(3)})$ and $B = (\mathrm{b}^{(1)}, \mathrm{b}^{(2)}, \mathrm{b}^{(3)})$ in F(R), we have the following comparison

(i) $A < B$ If and only if $\Re(A) < \Re(B)$

(ii) $A > B$ If and only if $\Re(A) < \Re(B)$

(iii) $A \approx B$ If and only if $\Re(A) = \Re(B)$

(iv) $A - B$ If and only if $\Re(A) - \Re(B) = 0$

A triangular fuzzy number $A = (a^{(1)}, a^{(2)}, a^{(3)})$ in F(R) is said to be positive if $\Re(A) > 0$ and denoted by $A > \tilde{0}$. Also if $\Re(A) > 0$, then $A > \tilde{0}$ and if $\Re(A) = 0$, then $A \approx \tilde{0}$. If $\Re(A) = \Re(B)$, then the triangular numbers A and B are said to be equivalent and is denoted by $A \approx B$.

For Symmetric Triangular Fuzzy numbers $a^{(2)} = a^{(3)}$

4. Arithmetic operations of symmetric triangular fuzzy number based on r-cut

The fuzzy number is fully and uniquely represented by its r-cut, since the r-cut of each fuzzy number are closed interval of real numbers for all r∈(0,1]. This enables us to define arithmetic operations on fuzzy number in terms of arithmetic operations on their r-cut.

Let A and B by arbitrary fuzzy numbers with the r-cut $A = \left[\underline{A}(\mathrm{r}), \overline{A}(r)\right]$ and $B = \left[\underline{B}(\mathrm{r}), \overline{B}(r)\right]$. Then the arithmetic operations between A and B are denoted by

(i) $A + B = \left[\underline{A}(r) + \underline{B}(\mathrm{r}), \overline{A}(r) + \overline{B}(r)\right]$

(ii) $A - B = \left[\underline{A}(r) - \overline{B}(r), \overline{A}(r) - \underline{B}(\mathrm{r})\right]$

(iii) $AB = H = \left[\underline{H}(r), \overline{H}(r)\right]$

Where $\underline{H}(r) = \min\{\underline{A}(r)\underline{B}(\mathrm{r}), \overline{A}(r)\overline{B}(r), \overline{A}(r)\underline{B}(\mathrm{r}), \underline{A}(r)\overline{B}(r)\}$

$$\underline{H}(r) = \max\{\underline{A}(r)\underline{B}(\mathrm{r}), \overline{A}(r)\overline{B}(r), \overline{A}(r)\underline{B}(\mathrm{r}), \underline{A}(r)\overline{B}(r)\}$$

(iv) $A / B = H = \left[\underline{H}(r), \overline{H}(r)\right]$

Where $\underline{H}(r) = \min\{\underline{A}(r)/\underline{B}(\mathrm{r}), \overline{A}(r)/\overline{B}(r), \overline{A}(r)/\underline{B}(\mathrm{r}), \underline{A}(r)/\overline{B}(r)\}$

$$\underline{H}(r) = \max\{\underline{A}(r)/\underline{B}(\mathrm{r}), \overline{A}(r)/\overline{B}(r), \overline{A}(r)/\underline{B}(\mathrm{r}), \underline{A}(r)/\overline{B}(r)\}$$

(v) $KA = \begin{cases} \left[K\underline{A}(r), \mathrm{K}\overline{A}(r)\right], & \text{if } \mathrm{K} \geq 0 \\ \left[\mathrm{K}\overline{A}(r), K\underline{A}(r)\right], & \text{if } \mathrm{K}<0 \end{cases}$

5. Mathematical formulation of a fuzzy transportation problem

Mathematically a transportation problem can be stated as follows:

Minimize

$$z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \qquad \text{------(1)}$$

Subject to

$$\left.\begin{array}{lll} \sum_{j=1}^{n} x_{ij} = \mathrm{a}_i & j = 1, 2, ..., \mathrm{n} & \\ \sum_{i=1}^{m} x_{ij} = \mathrm{b}_j & i = 1, 2, ..., \mathrm{m} & \\ x_{ij} \geq 0 & i = 1, 2, ..., m, & j = 1, 2, ..., n \end{array}\right\} \qquad \text{----(2)}$$

Where c_{ij} is the cost of transportation of an unit from the ith source to the jth destination, and the quantity x_{ij} is to be some positive integer or zero, which is to be transported from the ith origin to jth destination. A obvious necessary and sufficient condition for the linear programming problem given in (1) to have a solution is that

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j \qquad \text{------(3)}$$

(i.e) assume that total available is equal to the total required. If it is not true, a fictitious source or destination can be added. It should be noted that the problem has feasible solution if and only if the condition (2) satisfied. Now, the problem is to determine x_{ij}, in such a way that the total transportation cost is minimum

Mathematically a fuzzy transportation problem can be stated as follows:

Minimize

$$z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \qquad \text{------(4)}$$

Subject to

$$\left.\begin{array}{lll} \sum_{j=1}^{n} x_{ij} = \tilde{a}_i & j = 1,2,\ldots,\mathrm{n} & \\ \sum_{i=1}^{m} x_{ij} = \mathrm{b}_j & i = 1,2,\ldots,\mathrm{m} & \\ x_{ij} \geq 0 & i = 1,2,\ldots,m, & j = 1,2,\ldots,n \end{array}\right\} \qquad \text{----(5)}$$

In which the transportation costs $\tilde{c}_{ij}$, supply a_i and demand $\tilde{b}_j$ quantities are fuzzy quantities. An obvious necessary and sufficient condition for the fuzzy linear programming problem give in (4-5) to have a solution is that

$$\sum_{i=1}^{n} a_i \,\square\, \sum_{j=1}^{m} \tilde{b}_j \qquad \text{------(6)}$$

This problem can also be represented as follows:

	1	………	n	Supply
1	$\tilde{c}_{11}$	………	$\tilde{c}_{1n}$	a_1
. . . .	. . . .	. . . .	. . . .	. . . .
m	$\tilde{c}_{m1}$	……....	$\tilde{c}_{mn}$	a_m
Demand	$\tilde{b}_1$	………	$\tilde{b}_n$	

5.1 Theorem: 1 (Existence of fuzzy feasible solution)

The necessary and sufficient condition for the existence of a fuzzy feasible solution to the fuzzy transportation problem is $\sum_{i=1}^{n} a_i \,\square\, \sum_{j=1}^{m} b_j$

Proof: (Necessary condition)

Let there exists a fuzzy feasible solution to the fuzzy transportation problem given in (1)

Then $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \approx a_i$ and $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \approx \tilde{b}_j$. Therefore $\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} \tilde{b}_j$

(Sufficient condition)

Let as assume that $\sum_{i=1}^{n} a_i \square \sum_{j=1}^{m} \tilde{b}_j$. We have to distribute the supply at the i-th source in proportion to the requirements of all destinations.

Let $x_{ij} = \lambda \tilde{b}_j$, where λ_i is the proportionality factor for the i-th source. Since the supply must be completely distributed.

$$\sum_{i=1}^{n} x_{ij} \approx \lambda_i \sum_{j=1}^{m} \tilde{b}_j$$

Therefore $x_{ij} \approx \lambda_i \tilde{b}_j = \dfrac{a_i}{\sum_{j=1}^{n} \tilde{b}_j} . \tilde{b}_j$

$$\sum_{j=1}^{n} x_{ij} = \lambda_i \sum_{j=1}^{n} \tilde{b}_j = \frac{a_i}{\sum_{j=1}^{n} \tilde{b}_j} . \sum_{j=1}^{n} \tilde{b}_j = a_i$$

$$\sum_{i=1}^{m} x_{ij} = \frac{\tilde{b}_j}{\sum_{i=1}^{m} a_i} . \sum_{i=1}^{m} a_i = \tilde{b}_j$$

Which shows that all the constraints are satisfied. Since a_i and $\tilde{b}_j$ are positive, x_{ij} determined must be all positive. Therefore the fuzzy transportation problem yields a fuzzy feasible solution.

6. Procedure for solving Fuzzy Transportation Problem

We shall present a solution to fuzzy transportation problem involving shipping cost, customer demand and availability of products from the producers using Symmetric Triangular Fuzzy numbers.

Step 1: First convert the cost, demand and supply values which are all Symmetric Triangular Fuzzy numbers into crisp values by using the measure.

Step 2: we solve the transportation problem with crisp values by using VAM procedure to get the initial solution solution to get the optimal solution and obtain the allotment table.

Remark

A solution to any transportation problem will contain exactly (m+n-1) basic feasible solutions. The allotted value should be some positive integer or zero, but the solution obtained may be an integer or non-integer, because the original problem involves fuzzy numbers whose values are real numbers. If crisp solution is enough the solution is complete but if fuzzy solution is required go to next step.

Step 3: Determine the locations of nonzero basic feasible solutions in transportation table. There must be atleast one basic cell in each row and one in each column of the transportation table. Also the m+n-1 basic cells should not contain a cycle. Therefore, there exist some rows and columns which have only one basic cell. By starting from these cells, we calculate the fuzzy basic solutions, and continue until (m+n-1) basic solutions are obtained.

7. Numerical Examples

Consider the following fuzzy transportation problem

		DESTINATION				**SUPPLY**
	SOURCE	(2,3,3)	(2,3,3)	(2,3,3)	(1,4,4)	(0,3,3)
		(4,9,9)	(4,8,8)	(2,5,5)	(1,4,4)	(2,13,13)
		(2,7,7)	(0,5,5)	(0,5,5)	(4,8,8)	(2,8,8)
DEMAND		(1,4,4)	(0,9,9)	(1,4,4)	(2,7,7)	

Convert the given fuzzy problem into a crisp value problem by using the measure.

		DESTINATION				**SUPPLY**
	SOURCE	2.83	2.83	2.83	3.5	2.5
		8.16	7.33	4.5	3.5	11.16
		6.16	4.16	4.16	7.3	7
DEMAND		3.5	7.5	3.5	3.66	

Using VAM procedure we obtain the initial solution as

			2.5
3.5	0.5	3.5	3.66
	7		

Now using the allotment rules, the solution of the problem can be obtained in the form of Symmetric Triangular fuzzy numbers.

		DESTINATION				**SUPPLY**
	SOURCE				(0,3,3)	(0,3,3)
		(1,4,4)	(-2,1,1)	(1,4,4)	(2,4,4)	(2,13,13)

			(2,8,8)			(2,8,8)
DEMAND		(1,4,4)	(0,9,9)	(1,4,4)	(2,7,7)	

Therefore the fuzzy optimal solution for the given transportation problem is $x_{14} = (0,3,3)$, $x_{21} = (1,4,4)$, $x_{22} = (-2,1,1)$, $x_{23} = (1,4,4)$, $x_{24} = (2,4,4)$, $x_{32} = (2,8,8)$.

And the crisp solution to the problem is Minimum cost =84.33.

8. Conclusion

In this paper a simple method of solving fuzzy transportation problem (supply, demand, and cost are all symmetric triangular fuzzy numbers) were introduced by using ranking of fuzzy numbers. The shipping cost, availability at the origins and requirements at the destinations are all symmetric triangular fuzzy numbers and the solution to the problem is given both as a fuzzy number and also as a ranked fuzzy number.

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