ON QUASISIMILAR HYPONORMAL AND DOMINANT OPERATORS

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ABSTRACT–*In this paper we extend some of the properties of hypo normal operators to the class of dominant operators.*

Keywords:Bounded linear operator,Hyponormal operator, Dominant operator, Semi-normal operators, Subnormal operator, Quasisimilar and Quasiaffinity.

1, INTRODUCTION

In 17th century J.G.Stampfli and B.L.Wadhwa was introduced the Dominant operator. After that C.R.Putnam had generalized the every hypo normal operator is dominant. L.R.Williams and Mehdi Radjablipur was also extended the general properties. Youngoh Yang discussed some new characterization of dominant operators. Later on many authors extended with different aspects. Still some new results are lying unclaimed in the domain of dominant operators. In this paper we gave to two theorems along with Lemma.

HYPONORMAL OPERATOR

Let H be a Hilbert space and $L(H)$ denote the set of all bounded linear operator on H. An operator $A \in L(H)$ is called hypo normal if $A^*A \geq AA^*$ where A^* is the self ad joint operator of A. Now if A^* is a hypo normal operator, then $A^{**}A^* \geq A^*A^{**}$ but $A^{**} = A$ so that $AA^* \geq A^*A$

Hence A is semi-normal operator.

Thus if A^* is a hypo normal operator, then A is a semi normal operator. Similarly we can prove that if A^* is a semi normal operator, then A is hypo normal operator.

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If H_1 and H_2 are Hilbert spaces and $A: H_1 > H_2$ is a bounded linear operator having trivial kernel and dense range, then A is called a Quasiaffinity.

If $A_1 \in L(H_1)$ and $A_2 \in L(H_2)$ and there exists quasiaffinities $A: H_1 > H_2$ and $B: H_2 > H_1$ satisfying $AA_1 = A_2A \,\&\, A_1B = BA_2$ then $A_1 \,\&\, A_2$ are known as Quasisimilar.
If A is an operator, Let $\sigma(A)$ denote the spectrum of A, $K(A)$ denote the kernel of A and $R(A)$ denote the range of A. If $A \in L(H)$ and H is infinite dimensional Hilbert space. Let $\sigma_\theta(A)$ denote the essential spectrum of A.

Dominant Operator

An operator A is said to be dominant if
$R(T-\lambda) \subseteq R\left((T-\lambda)^*\right)$ for each λ in $\sigma(T)$, by theorem (1) of [3] it can be easily followed that every hypo normal operator is dominant operator.
Let A is an operator then $A = A_1 \oplus A_2$ where A_1 is normal and A_2 is pure that is if M is a reducing subspace of A_2 and A_2/M is normal, then $M = (0)$.
The operator A_1 is called the normal part of A and A_2 the pure part of A. Also if A is a dominant operator, M is invariant subspace for A and A/M is normal, then M reduces A[7]. Thus if A is a pure dominant operator then the point spectrum of A is empty. Normal parts of quasisimilar subnormal operator are unitarily equivalent.

Theorem (1.1): Suppose A_1 and A_2 are two quasisimilar dominant operators.
Let $A_i = N_i \oplus V_i$ on the Hilbert spaces $H_i \oplus K_i$ where N_i and V_i are normal and pure part respectively of $A_i, i = 1, 2$. Then N_1 and N_2 are unitarily equivalent and there exists bounded linear operators $A_0 : K_1 > K_2 \,\&\, B_0 : K_2 > K_1$ having dense range such that
$A_0V_1 = V_2A_0 \,\&\, V_1B_0 = B_0V_2$.
For proving the theorem (1.1) we shall need the following theorem and lemma.

Theorem (1.2): Let $H_1 \,\&\, H_2$ are Hilbert spaces, A_1 is normal operator in $L(H_1)$, A_2 is normal operator in $L(H_2)$, $A: H_1 > H_2$ is a bounded linear operator and $AA_1 = A_2A$. If A_2 is pure then $A = 0$. Let $P_0 = A_2/M$ and $A_0 : H_1 > M$ defined by $A_0z = Az \;\forall z \in H_1$.
Observe that P_0 is dominant, A_0 has dense range and $A_0A_1 = P_0A_0$.
Hence by theorem1 of [7] P_0 is normal. Thus lemma 2 of [7] implies that M reduces A_2.
Hence since A_2 is pure we have $M = (0)$ and as such $A = 0$

Lemma (1.1): Let H_1 and H_2 are Hilbert spaces; N_i is a normal operator in $L(H_i), i=1,2$
$A: H_1 > H_2 \& B: H_2 > H_1$
are bounded linear operators having trivial kernels and
$AN_1 = N_2 A \& N_1 B = BN_2$
then N_1 and N_2 are unitarily equivalent.

Proof: Let $M = \overline{R(A)}$ and $N = \overline{R(B)}$
Then lemma (4.1) of (4) implies that M reduces N_2, N reduces N_1, N_1 is unitarily equivalent to N_2/M and N_2 is unitarily equivalent to N_1/N. Thus by the theorem (1.3) of (5) N_1 and N_2 are unitarily equivalent.

Proof of theorem (1.1): There exist quasiaffinities
$A: H_1 \oplus K_1 > H_2 \oplus K_2 \& B: H_2 \oplus K_2 > H_1 \oplus K_1$
Such that $AA_1 = A_2 A \& A_1 B = BA_2$
Let $\begin{pmatrix} A_1 & A_2 \\ A_3 & A_4 \end{pmatrix}$ and $\begin{pmatrix} B_1 & B_2 \\ B_3 & B_4 \end{pmatrix}$ are the matrices of A and B respectively with respect to $H_1 \oplus K_1 \& H_2 \oplus K_2$
A matrix calculation shows that $A_3N_1=V_2A_3$ and $B_3N_2=V_1B_3$ .Thus theorem (1.2) implies that $A_3=B_3=0$. It follows that A_1 and A_2 have trivial kernels and a matrix calculation shows that$A_1N_2=N_2A_1$ and $N_1B_1=B_1N_2$.
Hence by lemma (1.1), N_1 and N_2 are unitarily equivalent.
We find that A_4 and B_4 have dense ranges and $A_4V_1=A_2V_4$ and $V_1B_4=B_4V_2$
This completes the proof.

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