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GRAPHS WITH METRIC DIMENSION TWO

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$\beta(G)$

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ABSTRACT—**In this paper, we discuss some characteristics of a graph due to the properties of distance partition and establish some results pertaining to the structure of a graph *G* with □□2□. Finally the graphs with □ 2 is characterized which is in fact proved in [15].**

I. INTRODUCTION

In this chapter, the distance partition of vertex set of graph *G* is defined, with reference to a vertex in it and with the help of the same, characterize the graphs with metric dimension two (i.e. □ □*G* □ □ 2). For a given graph G, there are a number of properties related to the distance between two vertices and have been widely studied by various authors.The result given in Proposition 1.1.4 was observed by Samir Khuiler et. al [8], and is an important tool in deriving several interesting results of the present chapter. The Corollary 1.2.5. owes to Samir Khuiler et. al [8] and Corollary 1.2.6. is due to Sooryanarayan [13] and Corollary 1.2.7. proceeds from Sooryanarayanan, Murali, Harinath [14].

1.1. Properties of Distance Partition

In this section, we discuss some characteristics of a graph due to the properties of distance partition.

Definition 1.1.1. Let *G* be a graph with vertex set *V(G)* and v be a vertex in it. Then $\{ V_0, V_1, V_2, . . ., V_k \}$ is called a *distance partition* of *V(G)* with reference to the vertex v if $V_0 = \{ v \}$ and V_t contains those vertices which are at distance *i* from v for $0 < i < k$, where *k* is the eccentricity of v in *G*. The sets Vo, Vj, V_2 ,..., V_k are called *distance partitite sets.*

Example 1.1.2. Look at the graph *G* given in the Figure 1.1.1. *v2 v4*

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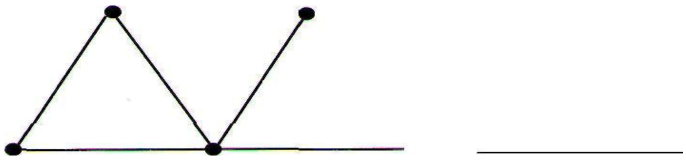


V1 *V3* *V5* *V6*

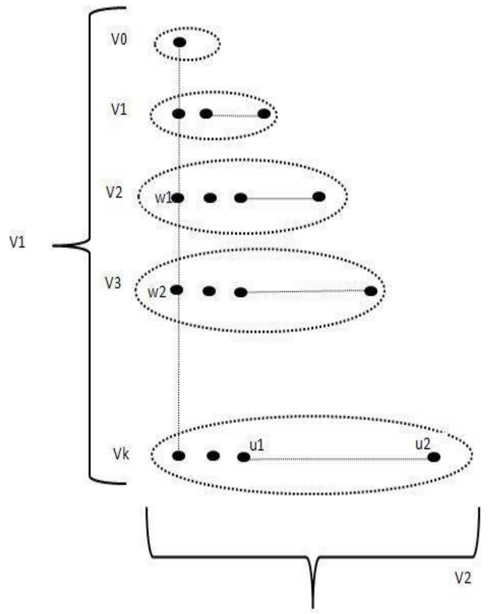
Figure. 1.1.1.

Let v □ $V (G)$. Then $V_0 = \{ v_1 \}$, $Vi = \{ v_2, v_3 \}$, $V_2 = \{ v_4, v_5 \}$, $V3 = \{ v_6 \}$ are called the distance partite sets of $V(G)$ with reference to the vertex v,.

Corollary 1.1.3. Let G be a graph with □ □G □ □ 2 and let $\{v_h\ \ v_2\ \}$ be a metric basis of G. Then every pair of vertices Wj and w>2 from different distance partite sets are resolved by at least v_7 and when uj and u_2 are from same distance partite set then v_2 resolves them.

Proof: Let u_1 and u_2 are from same partite set say Vj with reference to the vertex v_1. Since $d(u_1,v_1) = j = d(u_2,v_1)$ and $\{ v_1, v_2 \}$ be a metric basis of G, v_2 resolves u_1 and u_2. Now suppose w_1 and w_2 are from different distance partite sets say V_i and Vj respectively.
Since $d(w_1, v_1) = i$
and $d(w_2, v_2) = j$, w_1 and w_2 are resolved by v_1. This is shown in
Figure 1.1.2.

Figure 1.1.2.



Proposition 1.1.4. In a graph *G(V, E),* consider any three vertices *u,* v and *w* such that $u v \square E$. If $l = d(u, w)$

then $d(v, w)$ is one of $-1,$ and $+1.$

Corollary 1.1.5. Given any vertex $v \square V_i$. there exist at most three vertices in $V_{i \square 1}$` adjacent to v, where

$0 \square i \square e(v) \square 1$. Similarly there exist at most three vertices in $V_{i \square 1}$ adjacent to v when $1 \square i \square e(v)$.

1.2. Results Pertaining to the Structure of a Graph with $\square \square G \square \square 2$

This section establishes some results pertaining to the structure of a graph G with $\square \square G \square \square 2$. Further, let $\{V_0, V_1, V_2, \ldots, V_k\}$ be the distance partition of G with reference to the vertex v_1. The results of the Theorems 1.2.3 and 1.2.4 are due to Samir Khuller et. al [10] and a simple alternative proof using the concept of distance partition is given.

Theorem 1.2.1 For any vertex $v \square V_j$ there exists a shortest path of length between v_1 and v. In fact, a shortest path from v_1 to v contains exactly one vertex $w_1 \square V_j$ for $1 \square i \square j$, and the distance $d(w_1, v) \square j \square i$.

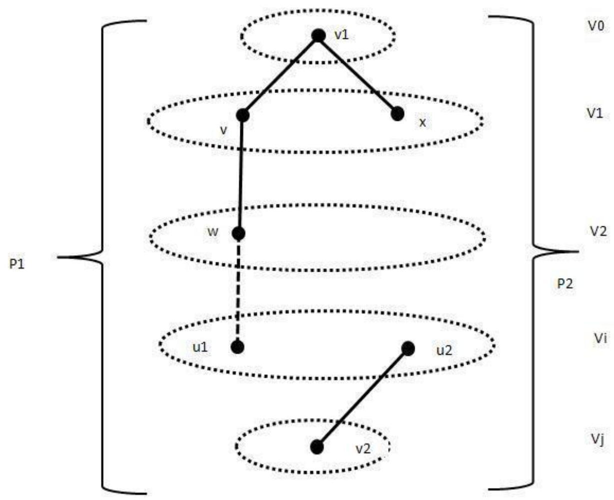
Proof. The first part of the theorem is immediate from the definition of distance partite set and $v \square V_j$. Note that if u_1 ,u_2 are adjacent and $u_i \square V_i$ for some $i \square 1$, then u_2 is in one of $V_{i \square 1}$, V and $V_{i \square 1}$. Suppose that a shortest path from v_1 to $v \square V_j$ of length j consists of more than one vertices $u_1, u_2 \square V_i$ where $1 \square i \square j$. Then the shortest path is of the form $v_1, w_1, \ldots, u_1, \ldots u_2, \ldots, v$. Since $d(v_1, v) = j$, $j = length(v_1 - u_1) + length(u_1 - u_2) + length(u_2 - v) > d(v_1, v_2) + length(v_2 - v)$. Since $u_1, u_2 \square V_i$ we have $d(v_1, u_1) \square d(v_1, v_2) \square i$ .So there exist a path v_1 to u_2 of length i. Hence we obtain a path $(v_1 \square u_2) \square (u_2 \square v)$ of length less than j from v_1 to u this contradicts

$d(v_1, v) \square j$

Theorem 1.2.2. If G is a graph with $\square \square G \square \square 2$ and metric basis $\{v_1, v_2\}$ then there exists a unique shortest path between v_1 and v_2.

Proof. Let $V_0, Vj, V_2, \ldots, V_k$ be the distance partite sets with reference to v_1 and $v_2 \square V_j$. By Theorem 1.2.1.

shortest path between v_1 and v_2 contains only one vertex from each distance partite set $Vo, Vj, V_2, \ldots, V_{j-1}$. Suppose that P_1 and P_2 are two shortest distinct paths between v_1 and v_2. Let Vi be the first partite set, while moving from v_2 to v_1, in which P_1 and P_2 pass through two distinct vertices u_1 and u_2 respectively. Then $d(v_2, u_1) = d(v_2, u_2)$ and hence u_1 and u_2 are not resolved by any of v_1 and v_2, a contradiction to the fact that $\{v_1, v_2\}$ is a metric basis of G, which is shown in Figure 1.2.



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Theorem 4.2.3. Let { v_1, v_2 } be a metric basis of G with □ □ G □ then degree of both v_1 and v_2 is less than or equal to three.

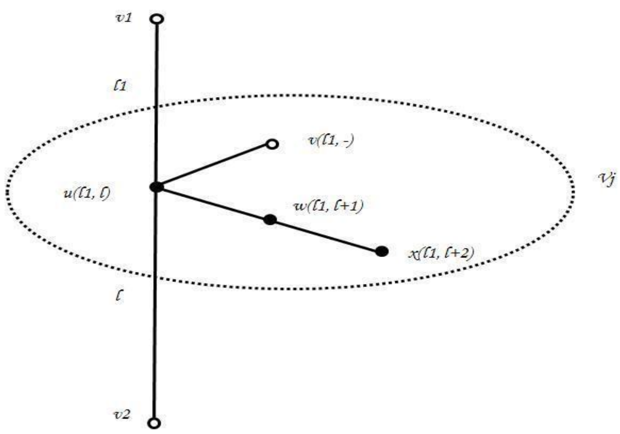
Proof. Let $d(v_1\ v_2) =$. Then any vertex adjacent to v_1 is at distance -1, *or* $+1$ from v_2. Since any pair of vertices that are adjacent to v_1 are not resolved by v_1, and are to be resolved by v_2, the distances from these vertices to v_2 are different. Hence the number of vertices adjacent to v_1 does not exceed three. In other words,

$deg\ v_1$ □ 3 .Similarly $deg\ v_2$ □ 3

Theorem1.2.4. Let { v_1, v_2 } be a metric basis G, where □ □G □ □ 2 .Consider distance partite sets $V_0, V_1, V_2, \ldots,$ V_k with reference to v_1.Any connected component of the graph induced by a distance partite set is a path and in fact, degree of any vertex in the graph induced by the distance partite set is at most two.

Proof. Let Vj be a distance partite set and C be a connected compound in the induced graph (Vj). Further, let u be among vertices in C such that □ $d(u, v_2)$ □ m in $_{v\ \square V}$ $d(v, v_2)$. Since v_2 resolves every pair of vertices in Vj,

the choice of u is unique. Then any vertex adjacent to u say w, is at distance $+ 1$ from v_2, any vertex adjacent to w, say x, is at distance $+ 2$ from v_2 and so on. In fact, for any v □ C $d(v_2, v) = d(v_2, u) + d(u, v)$. Thus the component C is a path and second part is trivial which is shown in Figure 1.2.2.



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Corollary 1.2.5. A graph *G* with □ □*G* □ □ 2 cannot have K_5 as a subgraph.

Proof. As diameter of K_5 is one, vertices of K_5 are to be there in at most two consecutive distance partite sets. Then at least one among possible two sets contain three or more vertices of Ks_5 which induces a cycle, which is not a path. Hence *G* cannot have K_5. This is shown in Figure 1.2.3. . □

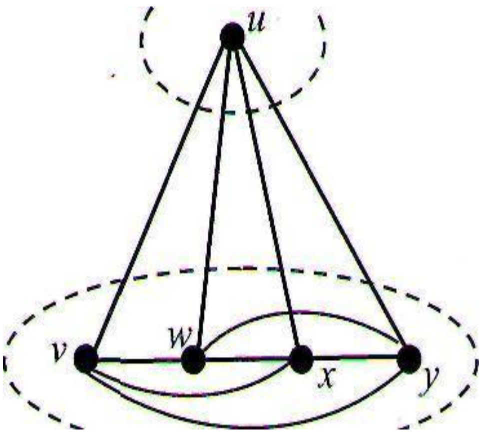


Figure 4.2.3.

Corollary 1.2.6. A graph *G* with □ □*G* □ □ 2 . Then for a triangle Tin *G,* if any, all the vertices of *T* cannot be at the same distance from v_1 or v_2.

Proof. Let *u, v, w* be the vertices of a triangle. If all the vertices are at distance-say *i* from v_i and all these vertices lie in the same partite set say V_i. Then all these vertices induces a cycle in the same partite set, a contradiction to Theorem 1.2.4.

Corollary 1.2.7. For any graph *G* with □ □*G* □ □ 2 the metric basis of *G* cannot have a vertex v of a subgraph K4 of *G.*

Proof. Let { v_1, v_2 } be a metric basis of *G* and v_1 □ $V(K_4)$. Consider the distance partite sets *Vo*, V_1 of *V(G)* with reference to v_1 .Then V_1 has the other three vertices of K_4 which induce a cycle, a contradiction to Theorem 1.2.4, which is shown in Figure 1.2.4.

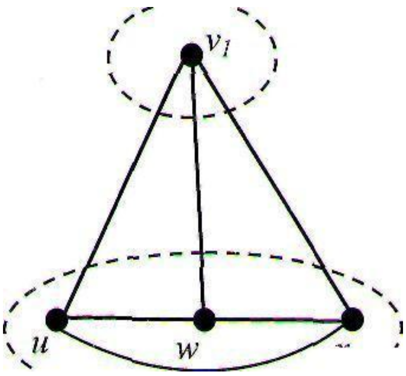


Figure 1.2.4.

Theorem 1.2.8. The maximum degree of any vertex in a graph *G* with □ □*G* □ □ 2 is eight and it is realizable.

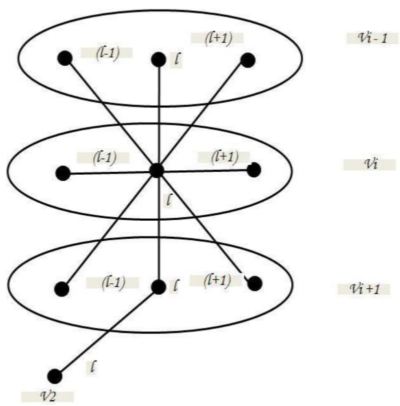
Proof. Let G be a graph with □ □*G* □ □ 2 and let $\{ v_1 , v_2 \}$ be a metric basis of *G*. By Corollary 1.1.5. and by

Theorem 1.2.4, given any vertex u □ V_i , it can be adjacent to at most three vertices each from $V_{i \square 1}$ and $V_{i \square 1}$

and at most two vertices from V_i. Hence the degree of *u* is at most eight. In the following Figure1.2.5, a graph *G* with □ □*G* □ □ 2 is

observed and a vertex of *G* having degree eight and all the vertices are labeled

with their distance from v_2.



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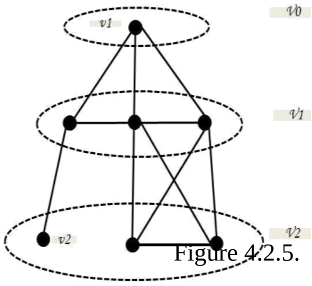

Figure 4.2.5.

Remark 1**.2.9.** The above Theorem gives an upper bound for degree of any vertex in a graph *G* with □ □*G* □ □ 2

Theorem 1.2.10. Let $(v_1 \; v_2)$ be a metric basis of *G*, where □ □*G* □ □ 2 . Then *G* cannot have K_5 — *e* as a

subgraph.

Proof: Since the graph induced by any distance partite set can have only components of paths and isolated vertices, vertices *of* K_5 — *e* are distributed as three (u_1, u_2, u_3) in one distance partite set, say V_i and other two $(u_4,$ $u_5)$ in an adjacent distance partite set, $V_{i \square 1}$ o r $V_{i \square 1}$ as shown in the Figure 1.2.6, in which case two of the three vertices u_1, u_2, u_3 are of degree three in K_5 - *e* and the remaining vertices are of degree four in K_5 - *e*. Without loss of generality, assume that u_1 and u_3 are of degree three and u_2, u_4, u_5 are of degree four in K_5 - *e*, as shown in the

Figure 1.2.6. Note that u_1, u_2, u_3 are pair wise resolvable by v_2 and so are u_4 and u_5. Now consider u_4 which is adjacent to all the remaining four vertices and let $d(v_2, v_4) =$ Further, as u_1, u_2 and u_3 are resolved pair wise

by v_2 and are adjacent to u_4, $d(v_2, u_j)$, $(j = 1, 2, 3)$ takes distinct values among ℓ + 1, ℓ, ℓ 1. Since u_5 is also

adjacent with all three vertices u_1, u_2 and u_3 , we get d(v_2 ,v)= ,a contradiction to the conclusion that u_4 and u_5

are resolved by v_2

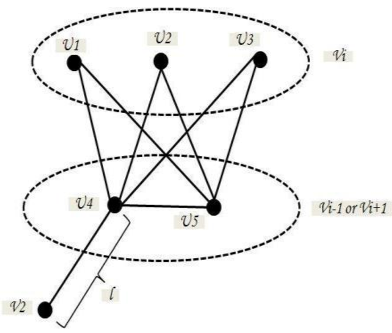

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Figure 4.2.6.

Remark. 1.2.11. It is clear that neither K_5 nor K_5 ~ *{e}* can be a subgraph of a graph with metric dimension two. So it is of natural curiosity how further smaller subgraph of K_5 can be excluded from being a subgraph of a graph from the class of graphs with metric Dimension two in the following Figure 1.2.7., we realize that K_5-2e could be a subgraph of some graph G with □ □G □ □ 2 .

Theorem 1.2.12. if G is a graph with □ □G □ □ 2 . Then G cannot have $K_{3,3}$ as a subgraph.

Proof: A graph G with □ □G □ □ 2 . Can have $K_{3,3}$ is present as sub graph and that there is a metric basis of size two. All vertices have been given distinct coordinates .Let the vertices of $K_{3,3}$ be $\{v_1, v_2, v_3\}$*and* $\{v_4, v_5, v_6\}$*.with edges going across from one set of vertices to the other .Among these six vertices, let* v_4 have the smallest first coordinates (a,b)..Vertices $\{v_1, v_2, v_3\}$ *must all have first coordinate ei8ther a or a+1* .

Suppose all three are $a+1$. The second coordinates must be

$\{b - 1, b, b + 1\}$ (in some order) this forces the second coordinates of vertices v_5.and v_6 to be b .There is no way assign distinct coordinates to vertices $\{v_4, v_5, v_6\}$..

Suppose all three are a. The second coordinates must $\{b - 1, b, b + 1\}$ (in some order). There are two vertices with coordinates (a, b).

Suppose vertices vj and v_2 have first coordinate a, and vertex v_3 has first coordinate $a + 1$. Vertices Vj and v_2 have their second coordinates $\{b - 1, b, b + 1\}$ in some order. Clearly the second coordinate of vertices v_5 and v_6 is b. There is no way to assign distinct coordinates to vertices $\{v_4, v_5, v_6\}$.

Suppose vertices v; has first coordinate a, and vertices v_2 and v_3 have first coordinate $a + 1$. The coordinates of the vertex v/ can be either $(a, b + 1)$ or

$(a, b- 1)$.

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Case 1. Coordinates of the vertex v/ is *(a, b + 1).*

In this case, the vertices v_2 and v_3 have to choose their second coordinates. The choices are *{b, b - 1}* or

{b, b + 1} or *{b + /, b - 1}*. We consider each case separately.

(i) The second coordinate of v_5 must be *b*. There is no choice for the first.

(ii) In this case vertices v_5 and v_6 have to pick from *{a, a + 1}* for the first coordinate and *{b, b + 1}* for the second coordinate. Since there are a total of four distinct choices and vertices v_1, v_2 and v_3 have used up three of them we cannot assign coordinates to v_5 and v_6.

(iii) The second coordinate of v_5 and v_6 must be *b*. There is no choice for the first.

Case 2. Coordinates of the vertex v_1 is *(a, b~ 1).*

In this case, vertices v_2 and v_3 have to choose their second coordinates. The choices are *{b, b -1}* or *{b, b + 1}*

or *{b + 1, b - 1}*. We consider each case separately.

(i) The choices for vertices v_5 and v_6 are *{a, a + 1}* for the first coordinate and *{b - 1, b}* for the second coordinate.

Since there are a total of four distinct choices and vertices v_1, v_2 and v_3 have used up three of them we cannot assign coordinates to vertices v_5 and v_6.

(ii) The second coordinate of v_5 must be b. There is no choice for the first.

(iii) The second coordinate of v_5 must be b. The first coordinate is forced to be a + 1. There is no choice for node v_6.

Theorem 4.2.13. Let $\{v1, v2\}$ be a metric basis of G, where $\beta(G) = 2$. Let $e(v1) = k$ and $|V(G)| = n$. Then eccentricity of the second resolving vertex v_2 is greater than or equal to $\left\lceil \frac{n-4}{k-1} \right\rceil, k \; 1$, where $[x]$ is the integer part of the number x.

Proof. Let $e(v_1) = k$ and $\{V_0, V_1, \ldots, V_k)$ be the distance partition of $V(G)$ with reference to v_1;. Then there is at least one distance partite set with number of vertices greater than or equal to $\left\lceil \frac{n-4}{k-1} \right\rceil, k \; 1$

Theorem 1.2.14. Let G be a graph with $\beta(G) = 2$ and $\{v_1, v_2\}$ be a metric basis of G. Let P be the Petersen graph. Then neither of v_1 and v_2 are in $V(P)$. Further, if eccentricity of any v_1; and v_2 is not more than three, then P cannot be a subgraph of G.

Proof: Consider distance partite sets $\{V_0, V_1, V_2, \ldots, V_k\}$ with reference to v_1. If $v_1 \in V(P)$ then V_2 consists of at least six vertices of $V(P)$ which induces a cycle in V_2. This is a contradiction. Hence $v_1 \notin V(P)$. Similarly $v_2 \notin V(P)$.Suppose that P is a subgraph of G and $e(v_2) = 3$. Now consider distance partite sets with reference to v_1. Then at most one Vj which contains v_2 may have four vertices and the remaining V_i have no more than three vertices. As $v_1 \notin V(P)$ and diameter of $P = 2$, $V(P)$ is distributed among three V_j's such that one having four vertices of $V(P)$

and other two having three each. This implies that $v_2 \in V(P)$ which is a contradiction.

Theorem 1.2.15. Let G be a graph *with* $\beta(G) = 2$ then there is no connected subgraph H of G such that

$d(H) < \sqrt{m} - 1$, where m is cardinality of $V(H)$.

Proof. Consider a metric basis $\{v1, v2\}$ of G, where $\beta(G) = 2$, and distance partition $\{V0, V1, V2, \ldots, Vk\}$ of

$V(G)$ with reference to among the basis elements, say v_1. Let H be any connected subgraph of G. Any pairs of

vertices, among vertices of H and in the same partite set, say Vj, are resolved by v_2. Since the distance between

any pair of vertices for $\{v_n \backslash v_n \in V(H) \cap V_j\}$ is not more than $d(H)$, $d(v_2, v_n)$ takes distinct values among

$\ell, \ell+1, \ldots, \ell+d(H)$ where $\ell = \min_{v \in H}\{d(v, v_2)\}$ .. So, the cardinality of $H \cap Vj$ is at most $d(H) + 1$.

Further, the vertices of H could be distributed among at most $d(H) + 1$ consecutive V_i' s. Hence the cardinality

of H is at most $(d(H) + 1)(d(H) + 1)$.

That is $m \leq (d(H) + 1)^2$, where m is cardinality of $V(H)$. Therefore $\sqrt{m} - 1 \leq d(H)$. This proves the

result.

Lemma 1.2.16. Let G be a graph with $\beta(G) = 2$ and $\{v_1, v_2\}$ be a metric basis of G. Further, let $\{V_0, V_1, V_2, \ldots, V_k\}$ be the distance partition of $V(G)$ with reference to the vertex v_1. Then every distance partite set can have at most two vertices more than the maximum possible cardinality of preceding distance partite set.

Proof. Consider a distance partite set V_i, and V_{i-1} has m vertices. Let $d(v_2, uj) = -i, -i+1, \ldots, -i+m^1$,

$(m^1 \leq m)$, where $u_j \in V_{i-1}$ .As every vertex in V_i, is adjacent to one or the other vertices in V_{i-1}, $d(v_2, w_i)$ where

$w_i \in Vi$ can take one of the distinct values $-i-1, -i, \ldots, -i+m^1+1$. Thus if $V,<. j$ has a maximum of

m^1+1 vertices then V_i has a maximum of m^1+1+2 vertices.

Theorem 1.2.17. Let G be graph with $\beta(G) = 2$ and $\{v_1, v_2\}$ be a metric basis of G. Further, let $\{V_0, V_,, V_2, \ldots, V_k\}$*be* the distance partition of $V(G)$ with reference to one of the vertices in the metric basis. Then

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maximum number of vertices in any distance partite set, say V_i for 0 □ i □ k *is* $(2i + 1)$.

Proof. Proof is by mathematical induction and induction is applied on *i*, the suffix of V_t for 0 □ *i* □ *k* . The result

is true for $i = 0$ and 1.Assume that the result true for *i*. That is, *Vi* has at most (2i +1) vertices. By the previous Lemma 1.2.16., $V_i + 1$ can have at most the vertices more than (2i+1) vertices. Hence $V_i + 1$ can have at most

$2i+3 = 2(i + 1) + 1$ vertices. By mathematical induction the result follows **for** any positive integer

i, whichis shownin Figure 1.2.8

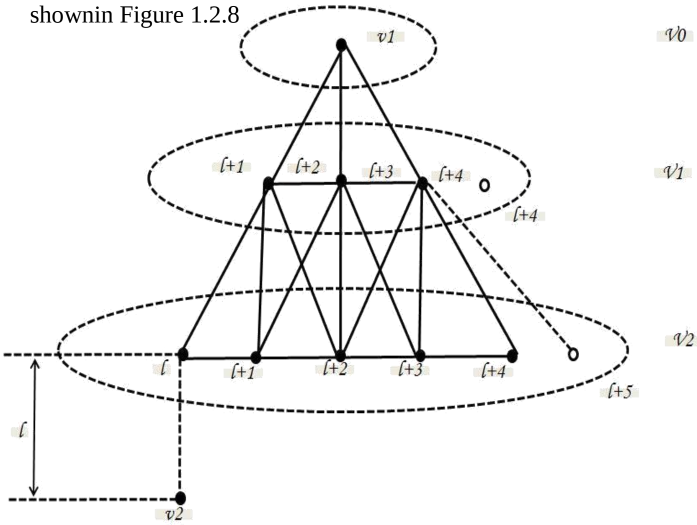


Figure 1.2.8.

1.**3. Characterization of Graphs with Metric Dimension Two**

In this section we determine the characterization of graphs with metric

dirnension two.

Theorem 1.3.1. Let *G* be a graph which is not a path with $V(G) = \{ v_1, v_2, . . . , v_n\}$ and $\{ V_{i0}, V_{i1} . . . , V_{ik}\}$ be the distance partition of *V(G)* with reference to the vertex v_i, where k_i is the eccentricity of v_i 1 □ *i* □ *n* . The metric dimension of G is 2 if and only if there exist vertices v_i and v_j such that $|V_{ik}$ □ $V_{jl}|$ □ 1 for every k and with 1 □ k □ $e(v_i)$ and 1 □ l □ $e(v_j)$.

Proof. Let G be a graph which is not a path with $V(G) = \{ v_1 v_2, \ldots, v_n\}$ and $\{ V_{i0}, V_{i1} \ldots, V_{ik}\}$ be the distance partition of $V(G)$ with reference to the vertex v_i, where k_i is the eccentricity of v_i 1 □ i □ n. Let □ □G □ □ 2. We have to prove that there exist vertices v_i, and v_j such that $|V_{ik}$ □ $V_{jl}|$ □ 1 for every k and with 1 □ k □ $e(v_i)$ and 1 □ l □ $e(v_j)$. Suppose not, for given v_p and v_r, $|V_{p_q}$ □ $V_{r_s}|$ □ 1 for some p_q and r_s implies that there exist at least two vertices, say $u1$ and $u2$ in V_{pq} □ V_{rs} such that $d(vp, u1) = d(vp, u2) = q$ and $d(vq, u1) = d(vq, u2) = s$ and hence u_1 and u_2 are not resolved by both v_p and v_r so, $|V_{p_q}$ □ $V_{rs}|$ □ 1 for all p_q and r_s implies no pair of vertices v_p and vr resolves $V(G)$, in other words □ □G □ □ 2.

Conversely if there exist v_p and v_r such that $|V_{p_q}$ □ $V_{r_s}|$ □ 1 for all p_q and r_s, then given any pair of vertices w_1 and w_2 from V(G) we have w_1 □ $V_{p_{q1}}$ □ $V_{r_{s1}}$ and w_2 □ $V_{p_{q2}}$ □ $V_{r_{s2}}$ where at least p_{q1} is different from p_{q2} or r_{sl} is different from r_{s2}. This implies that w_1 and w_2 are resolved by at least one of v_p and v_r. So □ □ G □ □ 2 and in fact, □ (G) □ 2 *as* G is not a path.

Illustration (i). Look at the graph G given in Figure 1.3.1. Let $V(G) = \{ v_1, v_2, v_3, v_4 \}$. Then

$V_{10} = \{ v_1 \}$, $V_{11} = \{ v_2, v_4 \}$ $V_{12} = \{ v_3 \}$,

$V_{20} = \{ v_2 \}$, $V_{21} = \{ v_1, v_3, v_4 \}$,

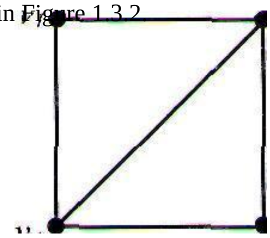
$V_{30} = \{ v_3 \}$, $V_{31} = \{ v_2, v_4\}$, $V_{32} = \{ v_1 \}$,

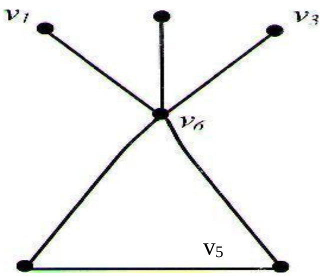
V_{40} = { v_4 }, V_{41} = { v_1, v_2, v_3} are the distance partite sets with reference to each vertex in *V(G)*.Since the vertices

v_1, v_2 □ V(G) such that $|V_{1k} \cap V_{2l}| \neq 1$ for every k and with $1 \leq k \leq e(v_1)$ and $1 \leq l \leq e(v_2)$, we have

$\Delta(G) \neq 2$.

(ii). Consider the graph *G* given in Figure 1.3.2





v_4

Figure 1.3.2.

Let *V(G)* = { $v_1, v_2, v_3, v_4, v_5, v_6$ }. Then

V_{10} = { v_1}, V_{11} = { v_6}, V_{12} = { v_2, v_3, v_4, v_5 },

V_{20} = { v_2 }, V_{21} = { v_6}, V_{22} = { v_1, v_3 v_4, v_5 },

V_{30} ={v_3}, V_{31} ={v_6}, V_{32} = { v_1, v_2, v_4, v_5 },

V_{40} = { v_4 } ,V_{41} = { v_5, v_6 }, V_{42} = { v_1, v_2, v_3 },

V_{50} = { v_5 }, V_{51} = { v_4, v_6 }, V_{52} = { v_1, v_2, v_3 },

V_{60} = { v_6 }, V_{61} = {v_1, v_2, v_3, v_4, v_5)

are the distance partite sets with reference to each vertex in *V(G)*.

Since no two vertices v_i ,v_j □ *V(G)* such that $|V_{ik} \cap V_{jl}| \neq 1$ for every *k* and with $1 \leq k \leq e(v_i)$ and

$1 \square l \square e($ v_j $)$,we have $\square(G) \square 2$.

References:

[1] Christopher Poisson and Ping Zhang. The metric dimension of unicyclic graphs. J.Combin. Math. Combin. Comput., 40:17–32, 2002.

[2] Buckley F and Harary F, Distance in graphs, Addison – Wesley (1990).

[3] Hartsfield Gerhard, Ringel, Pearls in Graph Theory, Academic press, USA (1994).

[4] Harary F and Melter R.A, On the metric dimension of a graph, Ars Combinatoria2 (1976), 191-195.

[5] F. Harary, Graph theory, Narosa/Addison Wesley (1969).

[6] Paul F. Tsuchiya, The landmark hierarchy; A new hierarchy for routing in very Large networks, ACM 0-89791-279-9/88/008/0035, 1988, page 35-42.

[7] Jose Cáceres, Carmen Hernando, Merce Mora, Ignacio M. Pelayo, Marìa. Puertas, Carlos Seara and David R. Wood, On the metric dimension of Cartesian product of graphs, arXiv: math.CO/0507527 v3 2 Mar 2006.

[8] Samir Khuller, Balaji Raghavachari, and Azriel Rosenfeld. Landmarks in graphs. Discrete Appl. Math., 70(3):217–229, 1996.

[9] Shanmukha B, Certain applications of number theory in graphs with emphases to networks. PhD. Thesis, 2003.

[10] Shanmukha B, Sooryanarayana B and Harinath K.S, Metric dimension of Wheels, FEJ. Appl. Math., 8(3)(2002), 217-229.

[11] Sooryanarayana B and Shanmukha B, A note on metric dimension, FEJ. Appl. Math., 5(3),(2001), 331-339.

[12]Sooryanarayana B, Certain combinatorial connections between groups, graphs and surfaces, PhD Thesis, 1998.

[13] Sooryanarayana B, On the metric dimension of a graph, Indian. J. Pure Appl.Math 29(4),(1998), 413 – 415[2].

[14] Sooryanarayana B, K.S. Harinath and R.Murali, Some results on metric

dimension of graph of diameter two,(communicated).

[15]Sudhakara.G and Hemanth kumar, Graphs with Metric Dimension Two-A Characterzation.World Academy

of science, Engineering and Technology,60(2009)