

---

## Analysis of Fracture parameters in Concrete by using ABAQUS software

---

**Ganesh Naidu Gopu**

*Assistant Professor, pace institute of technology and sciences, NH-5, near valluramma temple, ongole-523001, Andra Pradesh, India.*

*Gopuganeshnaidu@gmail.com*

**Abstract:** *It is commonly accepted that there is a size effect on the nominal resistances of quasi-brittle materials such as cementitious materials. This effect must be taken into account in the design of the ultimate behavior of concrete structures in order to avoid damage and crack openings. These parameters are frequently used to study the behavior of concrete and to characterize the durability of structures. Different theories exist in the literature to describe the size effect. Among them, we find the deterministic theory of Bazant where fracture energy is considered independent of the size and it is assumed that at peak load, the crack length is proportional to the size of the specimen. In this work, attention is paid to investigate numerically, the relationship between crack openings and length, and the size of the specimens. Various fracture parameters have also been studied by validating with the existing work for a concrete of grade M35 grade of concrete based on the RILEM standards. The present study shows the determination of fracture parameters of beams size-ranging from 100-400 mm using ABAQUS. Then analyzed the size-effect behavior of various fracture parameters obtained from different sizes of the beam.*

**Keywords:** Stress intensity factor, Strain energy release rate, Crack tip opening displacement, Fracture energy.

### 1. INTRODUCTION:

#### 1.1. Fracture in Concrete:

Fracture is a problem that society has faced for as long as there have been man-made structures. The problem may actually be worse today than in previous centuries, because more can go wrong in our complex technological society. Fracture mechanics is a branch of solid mechanics which deals with the behavior of the material and conditions in the vicinity of a crack and at the crack tip. The concept of linear elastic fracture mechanics has been well developed for more than past 40 years and successfully applied to metallic structures. Concrete is a heterogeneous anisotropic non-linear inelastic composite material, which is full of flaws that may initiate crack growth when the concrete is subjected to stress. Failure of concrete typically involves growth of large cracking zones and the formation of large cracks before the maximum load is reached. This fact several properties of concrete, point toward the use of fracture mechanics. Furthermore, the tensile strength of concrete is neglected in most serviceability and limit state calculations. Neglecting the tensile strength of concrete makes it difficult to interpret the effect of cracking in concrete.

Different non-linear fracture models are established for calculating the different fracture parameters. Those models are Frictional crack model, Crack band model, Size effect model, effective crack model, Two Parameter Fracture Model, Double G Fracture model, Double K fracture Model, Smeared Crack model, Discrete Crack model etc. The main fracture parameters can calculate from the models are Stress Intensity Factor, Crack Tip Opening Displacement and Strain Energy Release Rate etc.

#### 1.2. Size effect in fracture:

The influence of size effect on concrete structures has been a challenging issue during the recent past considering the fact that, the raw materials as well as type of concrete adopted. The size effect in quasi-brittle materials such as concrete is a well-known phenomenon and there are a number of experimental and theoretical studies. The size-effect is the decrease in nominal strength of geometrically similar structures subjected to symmetrical loads when the characteristic size of the structure is increased. There are two extremes of size-effect law as (i) strength criteria and (ii) LFM size-effect. The former yields no size-effect

whereas the latter shows the strongest size-effect i.e. nominal strength is inversely proportional to the square root of the structural dimension.

LEICESTER seems to have been first to investigate the effect of size on the structures made of metals, timber and concrete. In order to illustrate size dependence in a simple and dimensionless way, Hillerborg introduced the concept of characteristic length. As a unique material property, the characteristic length expresses fracture of concrete and concrete like materials, where the characteristic length ( $l_{ch}$ ) proportional to  $f_t^2$ . This means that brittleness increases with an increase in the strength of concrete, but it decreases with high fracture energy, according to fractious crack model (F.C.M).

#### 1.3. Introduction of FEM and ABAQUS:

The finite element analysis is a numerical technique. In this method all the complexities of the problems, like varying shape, boundary conditions and loads are maintained as they are but the solutions obtained are approximate. It started as an extension of matrix method of structural analysis. Today this method is used not only for the analysis in solid mechanics, but even in the analysis of fluid flow, heat transfer, electric and magnetic fields and many others. Civil engineers use this method extensively for the analysis of beams, space frames, plates, shells, folded plates, foundations, rock mechanics problems and seepage analysis of fluid through porous media. Both static and dynamic problems can be handled by finite element analysis. The FEM is a highly suited method for approximating the solution to the differential equation governing the addressed problem. Some popular Finite Element packages are STAAD-PRO, GT-STRUDL, NASTRAN, ABAQUS and ANSYS. Using these packages one can analyze several complex structures.

A popular enriched method is the so-called extended finite element method, abbreviated XFEM. The XFEM was implemented by Dassault Systemes Simulia Corp. [2010] in their latest version of ABAQUS 6.10, which puts the engineer in a position of being able to qualitatively estimate crack patterns, spacing and widths for arbitrary geometries. A complete ABAQUS analysis usually consists of three distinct stages: preprocessing, simulation, and post processing. Discrete crack modeling concept in ABAQUS aims at the

initiation and propagation of dominating cracks, whereas the smeared crack model is based on the observation, that the heterogeneity of concrete leads to the formation of many, small cracks which, only in a later stage, nucleates to form one larger, dominant crack.

Fanella and Krajcinovic [4] studied the effect of size on the rupture strength of plain concrete by focusing on the phenomena in the meso-scale of the material. Bazant and Kazemi [2] reformulated Bazant's size effect law applied to rock and concrete for the nominal stress at failure in a manner in which the parameters are the fracture energy and the fracture process zone length. From experimental as well as TPCM model they observed that, for three-point bend beams, TPFM predicts that the nominal strength decreases with increasing beam size, but approaches to a minimum constant value when sizes of the beam become very large. Tang, Shah, et al., [3] used Two Parameter Concrete Model (TPCM) to study the size effect of concrete in tension. Kotsovost and Pavlov [9] investigated the causes of size effects in structural-concrete experimentally and computationally. Their results demonstrated that the finite element package used can also provide a close fit to experimental values.

Rios, Jorge and Riera [8] conducted experiments as well as introduced the concept of Discrete Element Modeling (DEM) approach for studying the size effect in the analysis of reinforced concrete structures. In their work they have taken 4 different specimens by varying the dimensions of the beam and observed crack patterns under applied loading conditions for each of the four specimens. Alam, Kotronis, et al., [1] conducted experimental and numerical investigations on the influence of size effect on crack opening, crack length and crack propagation. Results obtained have not shown the accuracy in using the proposed model effectively for Size Effect phenomenon compared to that of experimental results. Muralidhara Rao, Gunneswara Rao [6] conducted tests on 45 beams of geometrically similar notched plain concrete (M25) specimens of different sizes. Fracture energy calculated using Work-of-fracture method was increasing with the increase in size of specimen and decreasing with the increasing notch depth ratios.

The size-effect relationships between FCM, SEM and TPFM were developed by Planas and Elices [5] (1990) that predicted almost the same fracture loads for practical size range (100 400 mm) of precracked concrete beam for TPBT geometry. Ouy ang et al. [10] (1996) established an equivalency between TPFM and SEM based on infinitely large size specimens. It was found that the relationship between CTODs and cf theoretically depends on both specimen geometry and initial crack length and both the fracture models can reasonably predict fracture behavior of quasi-brittle materials. Hanson and Ingraffea [7] (2003) developed the size-effect, two-parameter, and fictitious crack models numerically to predict crack growth in materials for three-point bend test. The investigation showed that if the three models must predict the same response for infinitely large structures, they do not always predict the same response on the laboratory size specimens. Roesler et al. [11] (2007) plotted the size-effect behavior of experimental results, numerical simulation using cohesive crack model, size-effect model and two parameter fracture model for three-point bend test specimens. From the analysis of results it is found that

the size-effect behavior calculated from SEM and TPFM resembles closely.

Shailendra Kumar and S.V.Barai [12] presented the numerical study on two parameter fracture model, effective crack model, size effect model, double-k and double-g models. They found that The critical stress intensity factors obtained using SEM, ECM, DKFM and DGFm appear to be close to each other with an error range of  $\pm 20\%$ , TPFM predicted the most conservative critical stress intensity factor. The crack-tip opening displacement at unstable fracture load predicted using TPFM was more conservative than that predicted using DKFM or DGFm by about in the range of 27-47%.

### 3. FINITE ELEMENT MODELING OF CONCRETE BEAMS:

#### 3.1. Material Properties and Geometry:

Standard specimens of three-point bending test as shown in Fig. 2 are developed in the present study. Effect of self-weight of the beam is also considered in the numerical model. The influence coefficients of the COD equation are determined using linear elastic finite element method. The same grade of the concrete taken by Shailendra Kumar and S.V. Barai [33] in 2012 is taken in the present investigation as M35. The direct local tensile strength of the concrete  $f_t$  is taken as 3.21 Mpa. The modulus of elasticity of concrete (E) is taken as 30 Gpa and fracture energy ( $G_f$ ) is taken as 0.103 N/mm. The value of Poisson ratio ( $\nu$ ) is assumed to be 0.18. For TPBT specimen of notched concrete beam with  $B = 100$  mm, size range  $100 \leq D \leq 400$  mm and aspect ratio(S/D) ranging between 3-6, the finite element analysis is carried out for determining the fracture peak load using fictitious crack model at initial crack length/depth ( $a_0/D$ ) ratios ranging between 0.2-0.5.

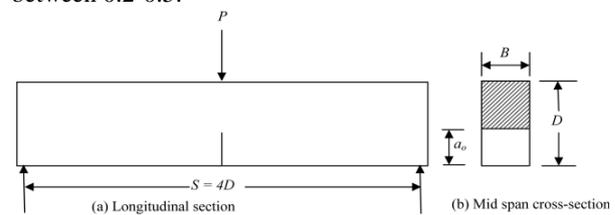


Figure 3.1: Three point bending test (TPBT) specimen geometry

In Fractious crack model the stress ranges between  $f_t$  and  $\frac{1}{3}f_t$ . An equation derived for calculating the stress  $\sigma$  from the stress-strain softening curve. The equation

$$\sigma = f_t \left( 1 - 3 \frac{w}{w_f} \right)$$

The terminal point of the softening curve is denoted as  $w_f$ , the crack width is noted by  $w$ . From these stress value we can found out the ultimate load (P) from the equation

$$P = \frac{1.072B(1-a_0)^2\sigma}{S}$$

Supports adopted were simply supported and the load is considered to be concentrated load acting at the centre of the beam. Notch depth was considered for analysis. The notch depths are taken as seam in vertical direction in the ABAQUS at the bottom of the beam exactly under the point load taken. Different notch depths were taken for the estimation of various fracture parameters. The crack face is

taken as the terminal line of the notch. The extension direction is taken as q vector which is taken before the notch face. Here the singularity is taken as 0.25.

**3.2. Mesh Size And Meshing Element:**

Generate the finite element mesh by choosing the meshing technique that ABAQUS/CAE can use to create the mesh, the element shape, and the element type. In the assigning section we can assign a particular ABAQUS element type to the model. Basic meshing is a two-stage operation: first seed the edges of the part instance, and then mesh the part instance. The number of seeds are based on the desired element size or on the number of elements that along an edge.

Seeding was done with three different element sizes. At the mid span of the beam very fine mesh has to be adopted, and mesh size gets reduced moving towards the supports. For  $L/D=4$  from support to a distance of D has been taken with course mesh. Then for 0.75D length has been taken medium mesh. At the mid span of length 0.25D has been taken very fine mesh. Structured hexagonal element has to be taken for meshing. See the figure below

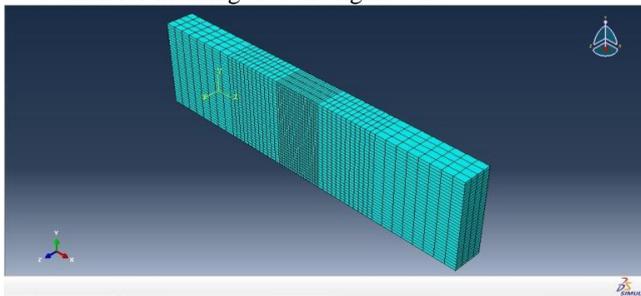


Figure 3.2: Seeding and meshing of the beam 300x1200

The quadratic reduced-integration elements available in ABAQUS/Standard also have hourglass modes. However, the modes are almost impossible to propagate in a normal mesh and are rarely a problem if the mesh is sufficiently fine. The  $1 \times 6$  mesh of C3D20R elements fails to converge because of hourglassing unless two elements are used through the width, but the more refined meshes do not fail even when only one element is used through the width. Quadratic reduced-integration elements are not susceptible to locking, even when subjected to complicated states of stress. Therefore, these elements are generally the best choice for most general stress/displacement simulations, except in large-displacement simulations involving very large strains and in some types of contact analyses.

**3.3. Output:**

The fracture parameters discussed are mainly are stress intensity factor (K), strain energy release rate ( $G_f$ ), crack tip opening displacement (CTOD) and length of fracture process zone. From the fracture analysis of the beam in ABAQUS we can get the J- integral which is called strain energy release rate in LEFM, stress intensity factor(K) for max strain energy release rate in mode I. generally the outputs are obtained in the .DAT, .FIL, .ODB and .PRT files. The stress intensity factor and strain energy release rate values are obtained in the .DAT file, and the stresses, strains, displacements and reaction forces are obtained in the .FIL file.

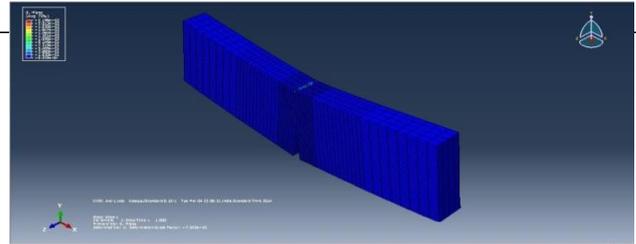


Figure 3.3: Stresses formed in the deformed beam in 3D

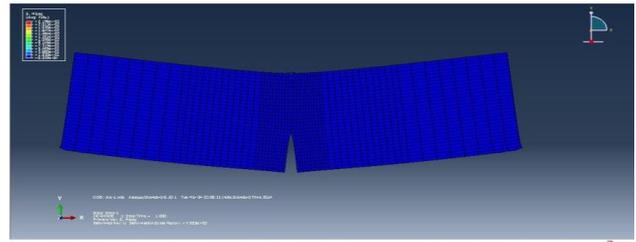


Figure 3.4: Stresses formed in the deformed beam in 2D

**4. RESULTS AND DISCUSSIONS**

**4.1. Validation and Comparison With Reference [12]:**

Stress intensity factor (K) was calculated from the finite element analysis using ABAQUS and CTOD value was calculated from equations. The obtained results of Discrete Crack Modeling (DCM) in table 2 are compared with results of Sailendra kumar and S.V.Barai [33].

**4.1.1. Size effect behavior of stress intensity factor of different models:**

Fig 4.1 shows the stress intensity factor obtained from different models with respect to the size of the beam for  $a/D=0.2$ . From the figures it clearly shows that the stress intensity factor varies with depth of the specimen. The stress intensity factor increases with increase in the depth of the beam. It shows the size effect occurs in the fracture of concrete. From the figures it can be observed that the stress intensity factor  $K_{IC}^e$  of effective crack model(ECM) shows the higher values. Double K and double G models values  $K_{IC}^{un}$  and  $\bar{K}_{IC}^{un}$  are almost equal with small variation and shows almost same size effect behaviour. The stress intensity factor  $K_I$  of calculated discrete crack model are nearly equal to the two parameter fracture model (TPFM) results  $K_{IC}^s$ . This means that TPFM and DCM predict the most conservative results of critical stress intensity factor at unstable failure. When the notch depth increases the K values obtained by calculated DCM slightly comes near to the K values obtained by TPFM. So the parameter K obtained from DCM was very similar to the parameter K obtained from TPFM.

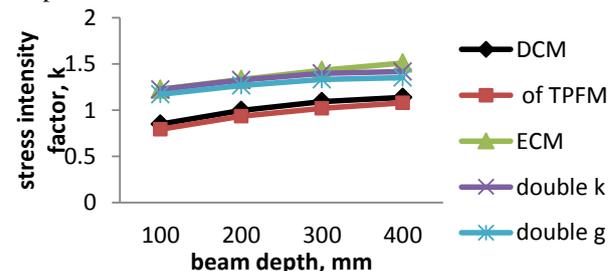


Figure 4.1: size effect behavior of stress intensity factor from different models for  $a/D=0.2$

**4.1.2. Size effect behavior of crack tip opening displacement (CTOD):**

The crack tip opening displacement (CTOD) of DCM obtained analytically was compared with  $CTOD_{CS}$  of TPFM and  $CTOD_C$  of Double K model obtained from the reference the figures 4.2 and 4.3 drawn for analysis.

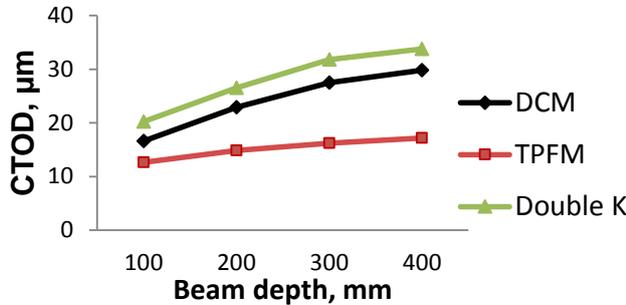


Figure 4.2: size effect behavior of crack tip opening displacement of different models for  $a/D=0.2$ .

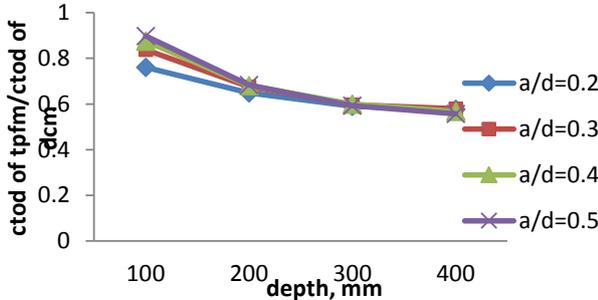


Figure 4.3: Relationship of the  $CTOD_{CS}$  and  $CTOD_D$  obtained between using TPFM and DCM.

From the fig 4.2 that the  $CTOD_D$ ,  $CTOD_{CS}$  and  $CTOD_C$  maintain a definite relationship with the specimen size for a given value of  $a/D$  ratio and they increase as the specimen size increases. It is also observed from the fig, the  $CTOD_{CS}$  and  $CTOD_C$  depend on the  $a/D$  ratio for a given specimen size. Here  $CTOD_{CS}$  of TPFM having lesser values compared to remaining values. The  $CTOD_D$  of DCM is nearer to the  $CTOD_C$  of Double k model. As notch depth increases the  $CTOD_D$  values obtained from DCM increase and slightly reaches to the  $CTOD_{CS}$  of Double K model. The  $CTOD_{CS}$  parameter values are more scattered particularly for smaller size of specimens when compared among the different  $a/D$  ratios whereas the  $CTOD_C$  values are more nearer and less scattered and appear to be in a narrow band for size-range 100-400 mm considered in the study.

A relationship between  $CTOD_{CS}$  and  $CTOD_D$  is presented in Fig. 4.3 in which the ratio  $CTOD_{CS}/CTOD_D$ , is plotted with respect to the depth of the beam D. It is seen from the figure that the ratio  $CTOD_{CS}/CTOD_D$  maintains a definite relationship with the specimen size and the ratio decreases as the specimen size increases. Neglecting the effect of  $a/D$  ratio, the mean values of  $CTOD_{CS}/CTOD_D$  for specimen sizes range 100 and 400 mm are determined and found to be 0.8425 and 0.57 respectively. It means that the predicted CTOD at critical load using DCM is conservative than TPFM and Double K model.

**4.2. Validation for Various Aspect Ratios of The Beam:**

Using the results and with reference to [35] the work has been extended to Plain concrete beams of different geometry with different aspect ratios  $L/d$  from 4 to 6 for

$a/D=0.2-0.5$  analyzed with discrete crack method in 3 dimensional finite element analysis using ABAQUS 6.10. The following fracture parameters have been calculated for different sizes of beams with different aspect ratios for different notch depths.

1. Stress intensity factor (K).
2. Crack tip opening displacement (CTOD).
3. Strain energy release rate ( $G_f$ ).

**4.2.1. Size effect behavior of stress intensity factor for various SENB beams of different sizes:**

The stress intensity factor values of various sizes of beams were calculated using three-dimensional finite element analysis of fractured beam.

**4.2.1.1. Variation of stress intensity factor with various beam sizes:**

Some graphs were drawn for analyze the size effect behavior of fracture parameter stress intensity factor (K). Here the stress intensity factor (K) varies with respect to the size of the beam. The figure 4.4 shows the stress intensity factor variation with the size of the beams.

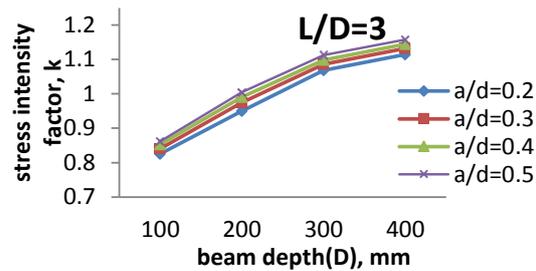


Figure 4.4: size of the beam vs stress intensity factor of SENB beams

The fig 4.4 clearly shows that stress intensity factor increases with the size of the beam. When observed the figures the line becomes flatter when the size of the beam increases. It means the stress intensity factor values become closer when the size of the beam increases. So it shows the size effect of stress intensity factor disappear when the size of the beam increases.

**4.2.1.2. Variation of stress intensity factor with various beam lengths**

figure 4.5 drawn that the stress intensity factor variation with the length of the beams.

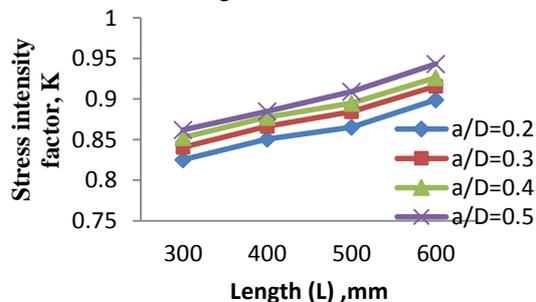


Figure 4.5: length vs stress intensity factor of SENB beams of depth 100mm

In fig 4.5 the lines were straight with constantly increasing values. It shows that the stress intensity factor varies lightly but constantly with the length of the beam. So it shows that that the size effect of stress intensity factor in

length is less but constantly moving with the length of the beam.

4.2.1.3. Variation of stress intensity factor with various depths of the beam:

Fig 4.6 shows the stress intensity factor variation with the depth of the beams.

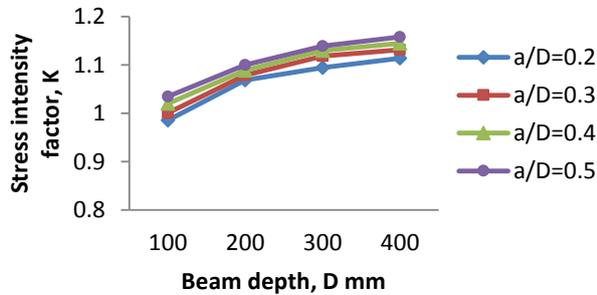


Figure 4.6: depth vs stress intensity factor in SENB beam of length 1200 mm.

The fig 4.6 showed clearly the stress intensity factor increases with the depth of the beam. The lines become flatter when the depth increases of the same length 1200 mm of the beam. So it clearly showed the size effect of stress intensity factor parameter constantly moving when depth of the beam increases for lesser beam lengths. For higher length beams the size effect of stress intensity factor disappears when the depth increases.

4.2.1.4. Variation of stress intensity factor with various notch depths of the beam:

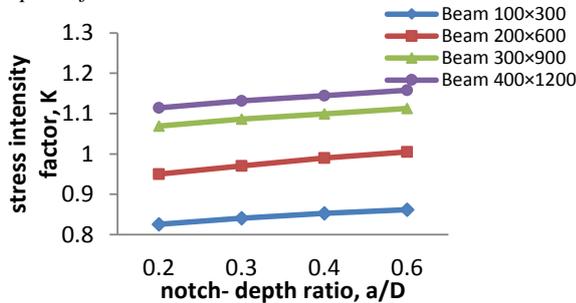


Figure 4.7: notch depth ratio vs stress intensity factor in SENB beams

Figure 4.7 showed the stress intensity factor variation with the notch-depth ratio. From graphs, it can observe that that stress intensity factor increases with the notch depth also. There was very small variation in the stress intensity factor in large notch depth ratio of the beam. So size effect of stress intensity factor is less when the notch depth of the beam increases.

4.2.2. Size effect behavior of various crack tip opening displacement (CTOD) for various SENB beams of different sizes:

Crack tip opening displacement calculated from stress intensity factor and strain energy release rate using numerical equations.

The crack tip opening displacement increases with the size of the beam. It means the Crack tip opening displacement becomes equal when the size of the beam increases. So it shows the size effect of Crack tip opening displacement disappears when the size of the beam increases.

The crack tip opening displacement increases with the length of the beam. It clearly means that the crack tip

opening displacement varies lightly but constantly with the length of the beam. So it shows that that the size effect of crack tip opening displacement in length is less but constantly moving with the length of the beam.

The crack tip opening displacement increases constantly with depth of the beam. The CTOD parameter variation is less when the depth increases of having higher lengths compared to lower lengths. So the size effect behavior of CTOD is less when the depth increases for the higher lengths of the beam compared o the lower lengths of the beam.

The CTOD increases with the notch depth also. A very small variation in the CTOD occurs when notch depth of the beam increases. So size effect of crack tip opening displacement is less when the notch depth of the beam increases.

4.2.3. Size effect behavior of strain energy release rate (G<sub>f</sub>) of various SENB beams of different sizes:

Strain energy release rate is called as j integral in non linear fracture mechanics. This parameter calculated directly from the finite element analysis using ABAQUS.

The strain energy release rate increases with the size of the beam. The strain energy release rate becomes equal when the size of the beam increases. So it shows the size effect of strain energy release rate decreases when the size of the beam increases.

The strain energy release rate increase with the length of the beam. the strain energy release rate varies lightly but constantly with the length of the beam. So it shows that that the size effect of strain energy release rate in length is less but constantly moving with the length of the beam.

The strain energy release rate increases with the depth of the beam of constant length having different notch depths. the variation of strain energy release rate is less when the depth increases of having higher lengths compared to lower lengths. So the size effect behavior of strain energy release rate is less when the depth increases for the higher lengths of the beam compared o the lower lengths of the beam.

The strain energy release rate increases with the notch depth also. It shows the lines are very flatter. It means there is very small variation in the strain energy release rate with notch depth ratio of the beam. So size effect of strain energy release rate is less when the notch depth of the beam increases.

5. CONCLUSIONS:

In the present study the size-effect analysis of various fracture parameters obtained from different sizes of the beam were calculated. The fracture parameters were determined from three-dimensional finite element analysis using ABAQUS 6.10 in Discrete crack modeling of size-ranging from 100-400 mm for which the input data were obtained from cohesive crack model. A comparative size-effect study was carried out using the possible fracture parameters from DCM, TPFM, SEM, ECM, DKFM and DGFM. In general, it was observed that all the fracture parameters were dependent on geometrical factor and specimen size. From present numerical study the following remarks can be highlighted.

- i. All fracture parameters obtained from numerical analysis of different sizes of beams are exhibiting the size effect phenomenon.
  - ii. The critical stress intensity factor obtained from numerical analysis gave proximate results when compared to the stress intensity factor obtained from TPFM.
  - iii. The crack tip opening displacement obtained from numerical analysis gave proximate results when compared to the Double K model and TPFM. But when continuously increasing sizes of the beam the crack tip opening displacement values of numerical analysis equals to the crack tip opening displacement of Double K model.
  - iv. When the size of the beam increases, the size effect behavior of fracture parameters are increases till some point and then remains constant which shows that the size effect behavior gets disappears.
  - v. When the length of the beam increases, the size effect behavior of all fracture parameters is small compared to other dimensions, but varies constantly.
  - vi. When the depth of the beam increases, the size effect behavior of all fracture parameters is large compared to other dimensions, but this behavior disappears while continuously increasing the depth.
  - vii. When the notch depth increases, then the size effect behavior of fracture parameters increases.
8. Jorge D. Riera, Roberto D. Rios. (2004), "Size effects in the analysis of reinforced concrete structures", *Engineering Structures*, Elsevier Ltd, 26, 1115-1125.
  9. Kotsovost. M. D., Pavlov. M. N., (1997), "Size effects in structural concrete: A numerical experiment", *Computers & Structures*, Elsevier Science Ltd. 64(1-4), 285-295.
  10. Ouyang. C. Shahs' and Tang. T, (1996) "Relationship between fracture parameters from two parameter fracture model and from size effect model", *Mater Struct* 29(2), 79-86.
  11. Roesler. J., Paulino, G.H., Park, K. and Gaedicke, C. (2007), "Concrete fracture prediction using bilinear softening", *Cement Concrete Compos.*, 29, 300-312.
  12. Shailendra Kumar and S.V. Barai, (2012), "Size-effect of fracture parameters for crack propagation in concrete: a comparative study", *Indian Institute of Technology, Kharagpur-721 302, India Computers and Concrete*, Vol. 9, No. 1 (2012) 1-19.

#### AKNOWLEDGEMENT

All authors are grateful to thank to the management of Sathyabama University, Chennai and BITS Hyderabad for the facility provided to complete the present study.

#### REFERENCES:

1. Ahmed Loukili., Panagiotis Kotronis., Syed Yasir Alam., (2013), Crack propagation and size effect in concrete using a non-local damage model", *Engineering Fracture Mechanics*, Elsevier Ltd, 109, 246-261
2. Bazant. Z.P, M.T. Kazemi., (1990), "Determination of fracture energy, process zone length and brittleness number from size effect, with application to rock and concrete" *International Journal of Fracture* 44: 111-131.
3. Chengsheng Ouyang., Tianxi Tang., Surendra P. Shah., (1992), "Fracture mechanics and size effect of concrete in tension". *Journal of Structural Engineering*, ASCE, 118(11), 3169-3185.
4. David Fanella1., Dusan Krajcinovic., (1988), "Size effect in concrete". *Journal of Engineering Mechanics*, ASCE, 114 (4), 704-715.
5. Elices.M and Planas.j, (1990), "Fracture criteria for concrete: mathematical validations and experimental validation", *Eng. Fract. Mech.*, 35, 87-94.
6. Gunneswara Rao.T.D, Muralidhara Rao.T., (2013), "Size effect of plain concrete beams: An experimental study", *International Journal of Research in Engineering and Technology*, 2(6), 1047-1055.
7. Hanson. J.H. and Ingraffea, A.R. (2003), "Using numerical simulations to compare the fracture toughness values for concrete from the size-effect, two-parameter and fictitious crack models", *Eng. Fract. Mech.*, 70, 1015-1027.